Math 3B   Quiz 3
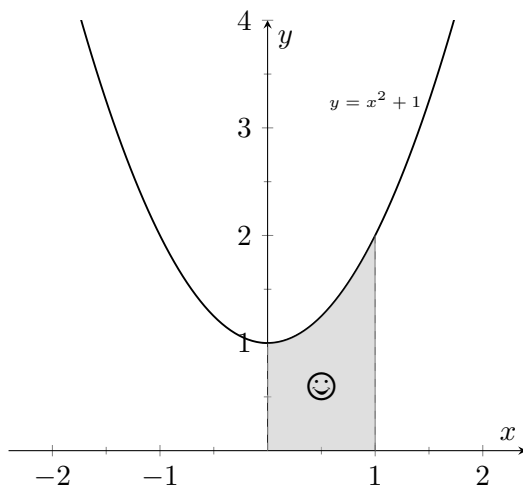
Name:_____   Section Day and Time:_____

Complete the following problems, making sure to SHOW ALL WORK. If you're stuck on something, CLEARLY EXPLAINING what you do know or what you would do will get you partial credit!

1. Consider the region bound between the functions $y = x^2 + 1$, $y = 0$, $x = 0$, and $x = 1$, which has been marked below.


Find the volume of the solid obtained by rotating the given region about the y-axis. You may use any method you'd like, but make clear what you've done.

There are a few ways we may go about this problem. One way is to split the y-axis from 0 to 1 and from 1 to 2, and to then use discs for the bottom half and washers for the top half. Another way is to use cylinders once. Cylinders are easier here, so let's do that, but it could be good conceptual practice to do the problem with discs and washers to make sure you get the same answer!

We begin by taking a vertical slice like in the figure below and rotating it around the y-axis to form

the shell of a cylinder.
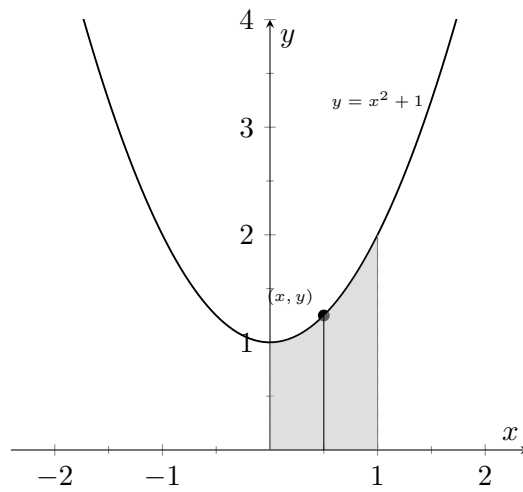
This cylindrical shell will have a radius of x and a height of y, giving us an area of $A = 2\pi xy$. Now, we integrate this from $x = 0$ to $x = 1$ to get

$$
\begin{aligned}
\text{Volume} &= \int_0^1 2\pi xy \, dx \\
&= 2\pi \int_0^1 x(x^2 + 1) \, dx \\
&= 2\pi \int_0^1 x^3 + x \, dx \\
&= 2\pi \left(\frac{1}{4}x^4 + \frac{1}{2}x^2 \right) \Big|_0^1 \\
&= 2\pi \left(\frac{1}{4} + \frac{1}{2} - 0 \right) \\
&= \frac{3\pi}{2}.
\end{aligned}
$$